Optimising laser absorption tomography beam arrays for imaging chemical species in gas turbine engine exhaust plumes

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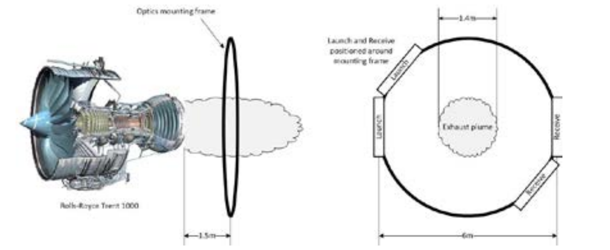
Introduction: *The FLITES (Fibre-Laser Imaging of Turbine Exhaust Species) project is an industry-academia consortium concerned with in-plume imaging of chemical species of interest to the aviation community, specifically CO_2 and Unburnt Hydrocarbons (UHCs), using laser absorption tomography. This poster presents the application of resolution matrices to the design of an optimised 126 beam, 6m absorption tomography array for imaging concentrations of CO_2 in the exhaust plume of a Rolls-Royce Trent 1000 gas turbine engine. The resolution matrix will be used to define a fitness value, which is a function of the beam configuration, and is minimised by the optimal beam arrangement. Constraints ensure that the optimised beam arrangement can be implemented in a real tomography system. Genetic algorithms are used to determine the optimal array design from the large problem set. Results for image reconstructions of a quasi-realistic phantom of the exhaust plume for each of the array designs are presented with indications of the reconstruction errors. From the results, conclusions are drawn on the suitability of applying resolution matrices to the design of beam arrays for real limited-data tomographic systems.*

Laser Absorption Tomography for Gas Turbine Engines

The FLITES project will realise the development of a 6m 126 beam laser absorption system for imaging concentrations of CO_2 in the exhaust plume of a Rolls-Royce Trent 1000 gas turbine engine. The geometric arrangement of the beam array and engine is illustrated below:



The optimum layout of sparse beam arrays is a long-recognised problem in limited-data hard-field tomography where reconstruction accuracy depends strongly on the arrangement of laser beams transecting the flow field. Existing beam array optimisation methods have centred on using the heuristic approach of distributing the beam layout in the sinogram space to increase the angular range of the beams (Terzija *et al* 2008). Recent work has sought to provide a structured mathematical approach to array design by understanding the underlying mathematical properties of the linear problem using resolution matrices (Twynstra and Daun, 2012).

Optimising Beam Array Designs Using Resolution Matrices

Resolution Matrices

Using Tikhonov regularisation, the regularised inverse can be expressed as (Daun, 2011):

$$\mathbf{A}^{\#} = \left(\mathbf{A}^{T}\mathbf{A} + \lambda^{2}\mathbf{L}^{T}\mathbf{L}\right)^{-1}\mathbf{A}^{T}$$

The **L** matrix is an approximation to the gradient function which adds the property of smoothness, or high-frequency noise filtering, to the image reconstruction by spanning the null space.

The reconstructed solution can then be determined by:

$$\mathbf{x}_{\lambda} = \mathbf{A}^{\#}\mathbf{b} = \mathbf{A}^{\#}\mathbf{A}\mathbf{x}^{exact} + \mathbf{A}^{\#}\delta\mathbf{b} = \mathbf{R}\mathbf{x}^{exact} + \mathbf{A}^{\#}\delta\mathbf{b}$$

The measurement vector can be expressed as $\mathbf{b} = \mathbf{b}^{exact} + \delta\mathbf{b} = \mathbf{A}\mathbf{x}^{exact} + \delta\mathbf{b}$. The resolution matrix, $\mathbf{R}$, is defined as the product of the sensitivity matrix and the regularised inverse. And $\mathbf{A}^{\#}\,\delta\mathbf{b}$ is the perturbation error associated with noise amplification in the reconstruction.

In an ideal experiment where $\delta\mathbf{b} = 0$, the reconstructed solution equals the exact solution where $\mathbf{R} = \mathbf{I}$. A fitness value, which seeks to minimise the distance between $\mathbf{R}$ and $\mathbf{I}$ and thus converge the reconstructed solution to the exact solution, can be expressed as:

$$F = \left\|\mathbf{R} - \mathbf{I}\right\|_{F}^{2}$$

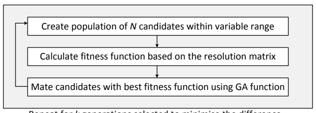
Optimisation Constraints

Design constraints are chosen to emphasise properties of the imaging problem and impose real physical limits on the practicality of implementing a given array design for a real laser absorption tomography system.

For real limited-data beam arrays for in-plume imaging in gas turbine engines three constraints are considered:
1. Array type; either regular or irregular
2. Problem symmetry; including both projection-symmetry and radial-symmetry
3. Variable optic position; for both launch and receive optics with a minimum distance constraint

Optimisation Method

Optimisation of the beam arrays will use a Genetic Algorithm (GA) to traverse the large search space. The basic algorithm for beam array optimisations incorporating the GA is shown below for N = 1000 and k = 10:

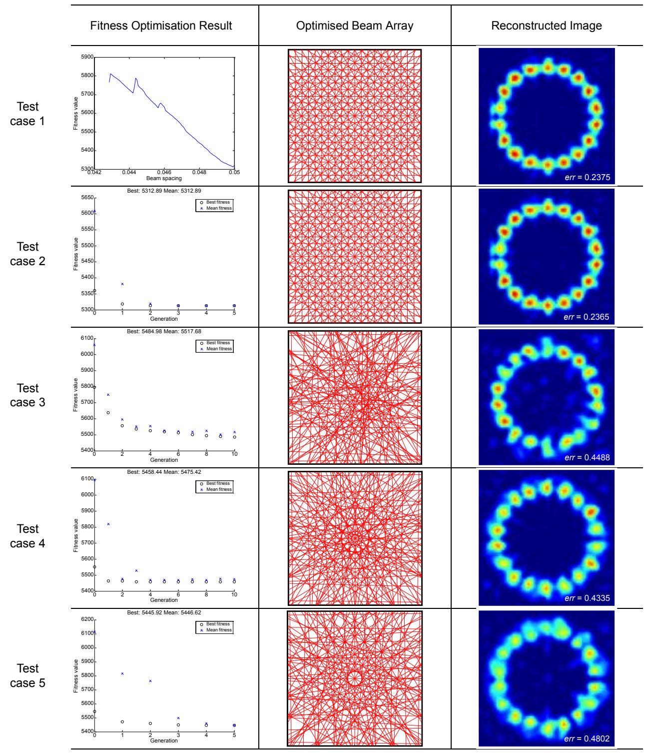


Repeat for k generations selected to minimise the difference between the mean and best fitness function

Optimisation Test Cases

Test case	Array type	Optimisation constraints	Variables
1	Regular	Fixed L/R position, single spacing for all projections	1
2	Regular	Fixed L/R position, single spacing per projections	6
3	Irregular	Fixed L/R position, variable L/R pairings	126
4	Irregular	Fixed L/R position, variable L/R pairings, projection and radial symmetry	10
5	Irregular	Variable L/R position, variable L/R pairings, projection and radial symmetry	20

Beam Array Optimisation Results



Discussion and Conclusions

- Resolution matrices can be used to design optimised beam arrays for laser absorption chemical species tomography.
- Design constraints ensure the design can be implemented in a laser absorption tomography system for imaging chemical species in the exhaust plume of a gas turbine engine.
- For regular arrays the widest beam spread in all projections gives the optimal arrangement.
- Imposed physical constraints limit the degrees of freedom in optimisations of irregular arrays.
- For irregular arrays, optimisation simulations are only sampling a small subset of the possible beam arrangements.
- Variable launch and receive positions gives a marginal improvement in fitness.
- Reconstructed images suffer from discretisation noise because of the coarseness of the 70x70 reconstruction grid.

The FLITES Consortium

References

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- Terzija, N., Davidson, J.L., Garcia-Stewart, C.A., Wright, P., Ozanyan, K.B., Pegrum, S., Litt, T.J., McCann, H., 2008. Image optimization for chemical species tomography with an irregular and sparse beam array, *Meas. Sci. Technol.* 19, 094007.
- Twynstra, M.G., Daun, K.J., 2012. Laser-absorption tomography beam arrangement optimization using resolution matrices, *Appl. Opt.* 51(29), p. 7059-7068.